Examiners' Report

Principal Examiner Feedback

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Pearson Edexcel International A Level

In Pure Mathematics (WMA13) Paper 01

Overview

The paper provided good access points for all students but also many challenging parts to stretch the better ones. Most students were able to attempt parts of all questions, though there were some omissions and relatively few completely question 10. This may have been more to do with not knowing how to proceed without being able to achieve the correct form of (c), however. Fewer questions seemed to be left completely blank in comparison with previous years, and those left blank were distributed across a few questions, not all question 10.

Many were able to re-start from incorrect work with a more successful attempt throughout the paper. The trigonometry questions seemed less chaotic than usual with very few making errors in identities. However, in question 3(b) many did not connect the two parts and were unable to make progress, which was surprising.

Practical questions such as interpreting 'q' in question 6 are still a weakness for most student, and this was the least scored mark of the paper. The final marks in question 9(c), the mark for the domain in question 2(b) and the interpretation in context in question 4(c) were the other most commonly lost marks.

Question 1

A familiar start, and students knew well what was expected of them in (a) and were able to carry out the iterative process for (b). It is noteworthy that the mark scheme for part (a) required more than just finding the values, demanding more than in previous papers and indeed there were a few students who failed to score the method mark for this reason. More commonly, though, a significant proportion lost just the A mark, mostly due to a failure to mention that f(x) is continuous. Most used separate < and > inequalities to consider the signs, though a few used f(1)f(2)<0, but both were acceptable. Also notable was that a number of students left the evaluations as expressions rather than decimals — although permitted, the signs of such expressions are not always clear from the expression, so numerical evaluation is advised.

Part (b) was well done. Some students showed substitution, some not, to reach 1.5105, and only rarely 1.7340 was not achieved. Students who ploughed through a page of working (to several decimal places), using up a lot of time when they could simply have used their calculators. The most common error in this part was insufficient accuracy, with 1.511 and 1.734 seen relatively often, scoring M1A0A1.

Question 2

This question was widely accessible with over three quarters of students able to score 7 or 8 marks of the 8. The third of students who lost only one mark typically omitted or made error with the domain of the inverse function $\mathrm{f}^{-1}(x)$ in part (b).

In part (a), students understood the notation and were successful in substituting $x=6$ into the function $\mathrm{f}(x)$ gain $\mathrm{f}(6)=\frac{9}{2}$ and then substituting this answer again into $\mathrm{f}(6)$, although a number of students found the composite function first before substituting. This latter was more prone to error, though typically was carried out correctly.

In part (b), most students replaced x with y and then made y the subject. The accuracy mark was generous in allowing the inverse notation as y, f^{-1} or $\mathrm{f}^{-1}(x)$ and therefore $y=\frac{4x+3}{x-1}$ gained both marks. Students proceeded from $y=\frac{x+3}{x-4}$ to then make x the subject before swapping variables sometimes lost the answer mark by not actually swapping, or not rewriting with the acceptable inverse notation as stated above. Occasional algebraic errors were made when simplifying, but generally the algebraic skills shown were strong. As noted above, a significant proportion of students lost the final mark for this section as they did not consider the domain of $\mathrm{f}^{-1}(x)$ at all, with $x>4$ or $x\neq 4$ also often seen as incorrect answers. Students should be reminded that the domain is required when defining a function.

In part (c), the majority of students gained the first mark by writing $\left(\frac{x+3}{x-4}\right)^2+5=7$ and then typically proceeded in one of two ways. The first approach was to manipulate by expanding, simplifying and forming a 3-term quadratic. Sign errors were accepted for the method marks, but a disappointing number of students used $(x+3)^2=x^2+9$ and could gain no further credit. Expanding and simplifying the quadratic was generally done well, leading to $x^2-22x+23=0$. A relatively common error was to factorise to $(a-23)(a+1)$ but such attempts were able to score the method mark. An exact answer of $x=11+7\sqrt{2}$ was required but if the other root of $x=11-7\sqrt{2}$ was stated, this was ignored as it was outside the domain.

The second, equally common, approach was to solve via $\left(\frac{x+3}{x-4}\right)^2+5=7\Rightarrow\frac{x+3}{x-4}=\pm\sqrt{2}$ and then rearrange to give $x=\frac{4\sqrt{2}+3}{\sqrt{2}-1}$ which was of an acceptable form. Many rewrote to give the final answer of $x=11+7\sqrt{2}$, though this was not necessary, and many also considered if $\frac{x+3}{x-4}=-\sqrt{2}$, also not necessary as the final answer was outside the valid domain of $x>0$. Attempts using the inverse function were rare, but decimal approximations were occasionally given, and these lost the final mark if the exact answer was not seen elsewhere.

Question 3

This question proved more challenging than expected with many students not making the connection between parts and so making little progress in part (b). Only a small proportion were able to score full marks, and nearly as many scored no marks at all, though not for lack of attempt. There was a full range of marks gained by students.

In part (a) most students made a full attempt, though the accuracy mark was quite discriminating and often lost due to missing steps at the start of the solution (lack of $\cos 2A$ = or LHS =, or no sight of $\cos A\cos A-\sin A\sin A$). The overarching process of needing to apply the compound formula and the Pythagorean identity was shown by many, so scoring the method, but the accuracy of detail in this trigonometric proof was sometimes lacking. However, there were a few cases of circular reasoning noted, where the Pythagorean identity was not used, but instead repeated use of the double angle formulae was used to replace the $\sin^2 A$ term – such attempts were not able to score the method mark as they relied on assumption of the given identity to prove the given identity and students ought to realise this is not acceptable.

Despite the "hence" in the question, some students struggled to make meaningful progress with part (b). Having been given the result needed, it surprising that many chose to ignore this and there were many attempts leading to $\cos^3 3x$ or $\sin^3 3x$ integrals, as well as some attempts at other incorrect methods. Those who did realise the identity was needed often scored both method marks, though there were numerous algebraic slips also made, with the process of substituting limits being well understood. Some again here gave decimal answers rather than heeding the instruction to find an exact answer and if no intermediate working was shown would lose the method as well as the accuracy mark at the end.

Question 4

The first of two context questions, the main difficulty with this question did lie in understanding and dealing with the context. There was access for all students with on very few unable to score, but equally few were able to attain full marks with the accuracy in part © being the most likely to drop. In part (a), nearly all students were able to find the value for A = 93 correctly, and those who didn't seldom many any progress in any parts.

Although most students were able to earn some credit in part (b) it was not a large majority as there were a significant number of students who gained no marks due to the substitution of N = 100 000, rather than N = 100, leading to a negative value for the exponential (so an unsolvable equation). Of those who set up the correct equation initially a few lost the final accuracy mark through premature/incorrect rounding, but most went on to find the correct value for I to the required level of accuracy, and so, gained full marks.

Part (c) was also mixed, though here a greater majority of students understood that they needed to differentiate and substitute t = 7 to find the rate of change in sales. However, many others persisted in simply substituting into the equation without differentiating, or finding two values and taking the difference over time. Very few students gained the accuracy mark, as they needed to remember that dN/dt was still in thousands. Having found an answer of 4.73 they needed to interpret it as 4,730 or 4.73 thousand, but the vast majority who achieve 4.73 left this as the answer.

Part (d) likewise was answer with mixed success, with many not engaging with the facts and trying to give a generic answer. Those who did relate to the question either compared the maximum possible sales that the model was able to calculate with the expected sales, often just stating the maximum value, or to use the model with the expected sales figure and reach a contradiction from an unsolvable equation. Some examples of answers that did not earn any credit here include those that referred to people's buying habits/whether or not they needed an upgrade, or incorrect values such as the maximum sales would be 125.

Question 5

Access in this question was generally good as long as students could get started, with over a half of the students able to score either 6 or 7 marks, but about a quarter of students scored either no marks, or just one mark.

The given form for the answer in (a) was an aid but did mean some students were able to deduce the "correct answer" from incorrect or unconvincing work. Most students identified the need to use the quotient rule, but many struggled to get the derivative fully correct because of a failure to use chain rule correctly (or at all) when differentiating $\ln(x^2 + k)$, resulting in a "constant" B of $1/x$. Such answers were then unable to score further marks due to being unable to solve the resulting equation. Occasional cases of the numerator terms with wrong way round, or missing the square in the denominator, also occurred. Completely incorrect solutions tended to be those who did not differentiate the ln term to a correct $\frac{..}{x^2+k}$ form at all.

Part (b) was generally done well by those who reached an answer (whether correctly or by virtue of knowing the form from the question) of the required form in (a). Most of these were able to solve the ln equation by a correct means, usually resulting in at least one correct follow through solution, though some neglected to take the negative square root, while others overlooked the $x = 0$ solution (sometimes the only mark dropped in the question). It was rare to see any students trying to use the denominator to find solutions.

The context of the upper limit in part (c) was better understood than the contextual parts of questions 4 and 6. However, it was more common for students to write an inequality in © than a statement of the upper limit, and students were penalised for including equality in such cases as this was a misunderstanding of the situation. Again, this was commonly the sole mark lost in the question.

Question 6

This question is another that provide good access, with the majority scoring in the 3–5-mark range, but very few students accessed the final mark of the question. The context, particularly for the q value, was not well understood.

In part (a) a lack of units on the answer was most common error, with students not making a distinction between the value of S, and the area of the sea floor covered by reefs. Most were able to substitute $t = 2$ into the equation and solve, only very few showed no method.

Part (b) was answered with more mixed levels of success, with the value of q being the most discriminatory part. Most had the idea of needing to raise to a power base 10, some just directly writing the constants as powers. The difficulty lay in sorting out the split of the terms into two terms, with sign error being quite common. Also, many did not give the complete equation asked for, but gave just statements for the values of p and q.

The interpretation for the constant q in part (c) proved to be very discriminating. There was a wide variety of attempts with very unclear understanding shown and the resulting tightness of the mark scheme meant very few could access the mark. The biggest misunderstanding was in the failure to recognise the value as a yearly proportional change, with many answers stating in effect an absolute change per year. There were many who gave the interpretation of p instead, the starting value for the area of coral on the sea floor being an easier concept to understand and one in which students may be well drilled. Seldom was the lack of mention of "coral" or "sea floor" or "each year", or their equivalents, the only reason for the loss of the mark, though the insistence for these was a great aid to markers in quickly screening out results that could not access the mark.

Question 7

This question proved accessible to the vast majority of students but also provided plenty of discrimination over the ability range with a wide spread of marks scored for this question with 0, 2 or full marks being the most common scores. Parts (a) and (b) perhaps provided most access with more failing to gain credit in part (c).

In part (a), the vast majority substituted $x = 0$ and found the value 9, with around half able to gain full credit in stating $0 \le \mathrm{f}(x) \le 9$ or $\mathrm{f}(x) \in [0,9]$. Other acceptable notation for the required mark was to use f or y in place of $\mathrm{f}(x)$. Errors made were usually in excluding the lower limit of $\mathrm{f}(x) = 0$, using strict inequalities or thinking that 9 was the lower limit.

In part (b), the product rule was well known but many students were unable to differentiate e^{-x^2} to an acceptable form of $Ax\mathrm{e}^{-x^2}$. Unlike in question 6 where the first mark was allowed without correct use of the chain rule, here they needed to differentiate $(2x^2 - 3)^2$ to a required form of $Bx(2x-3)$, in order to access the first mark (and hence any marks) in (a). Many, however, succeeded in this to achieve a result of $\mathrm{f}'(x) = -2x\mathrm{e}^{-x^2}(2x^2-3)^2 + 8x\mathrm{e}^{-x^2}(2x^2-3)$. Students who then proceeded to expand the bracket $(2x^2-3)^2$ to give $(4x^4 - 12x^2 + 9)$ usually also expanded the second term to give $(16x^3 - 24x)$ and were often then unsuccessful in factorising their answer to the required form. Those who were able to factorise by considering the factors $2x\mathrm{e}^{-x^2}(2x^2-3)$ generally gained the second method mark and then proceeded to the correct answer. If errors were made it was usually due to sign errors when expanding and collecting like terms in $4-(2x^2-3)$. Attempts to use the quotient rule were rare, and more prone to error.

In part (c), many students failed to score any marks, either because they had been unable to arrive at a suitable answer in part (b), or, having proceeded to find roots of their $\mathrm{f}'(x) = 0$ giving $x = 0$, $x = \sqrt{\frac{3}{2}}$ and $x = \sqrt{\frac{7}{2}}$, they then failed to find the corresponding value of $\mathrm{f}(x)$. Those students who were successful in reaching $\mathrm{f}\left(\sqrt{\frac{7}{2}}\right) = 16\mathrm{e}^{-\frac{7}{2}}$ or 0.483 generally proceeded to the final answer of $16\mathrm{e}^{-\frac{7}{2}} < k < 9$, although some did not realise they needed to use the 9 found in part (a). Those that used the approximation 0.483 lost out on the final accuracy mark. Occasionally only 0.48 was seen, which meant that neither accuracy mark was scored in this part – students should be reminded that any rounding should be to 3 s.f.

Question 8

This was another question with a widespread across all marks seen, provide access at all levels, but proving a good discriminator across all grades. Less than one in four students were able to gain full marks, but most were able to score 2 or more. A mixed level of ability in trigonometric work was seen in the trigonometry questions on this paper.

In part (a) most students were able to score the first two method marks for correct identities for dealing with the $\text{cosec}^2 2\theta$ and attempting to apply a double angle identity. The majority converted the $\text{cosec}^2 2\theta$ into $\frac{1}{\sin^2 2\theta}$, though some used the identity $\text{cosec}^2 2\theta = 1 + \cot^2 2\theta$ first to then write it in terms of both $\cos^2 2\theta$ and $\sin^2 2\theta$. At the same time, many were able to apply at least one correct double angle formula (up to allowing for sign errors), though not always the one that leads most directly to the result.

The third method mark was considerably harder to achieve, requiring all identities used to be fully correct and simplification to occur enough to reasonably believe that the proof could be finished from this point, and many responses scored only the first two marks as a result. The most successful attempts were those who made a careful choice over which form to use of the double angle formula for $\cos 2\theta$, with those choosing $1 - 2\sin^2\theta$ being the most successful. A common error which cost this mark was attempting to apply $\sin 2\theta = 2\sin\theta\cos\theta$ to the sin 2θ term in the denominator but not showing $(2\sin\theta\cos\theta)^2$ explicitly first and then failing to square the constant. This meant they had not shown evidence of a correct identity, so the method mark was lost as well as the accuracy mark becoming impossible. In the numerator, sign error dealing with the removal of the bracket around "$2 - 2(1 - 2\sin^2\theta)$" was the most common error, but these could still allow the third method mark to be scored even if they cost the accuracy.

Of those who scored up to the third method mark, the most common reason for not achieving the final accuracy mark was the omission of the $\sec^2\theta$ step before reaching $1+\tan^2\theta$.
A small number of students did attempt to work from both sides, usually with limited success due to failing to provide an appropriate conclusion for their work.

Part (b) of the question was considerably better attempted. The vast majority of students successfully substituted the result from part (a) and proceeded to a three-term quadratic in either $\sec\theta$ or $\cos\theta$ which they solved to reach a correct value for $\cos\theta$ and so achieved the first pair of method and accuracy marks. A small number who reached $\sec\theta = 4$ were not able to proceed from there to find a value for $\cos\theta$ and some indeed used an incorrect identity such as $\sec\theta = \frac{1}{\tan\theta}$, losing the accuracy. The alternative approach via $\tan\theta$ was less common and prone likely to go wrong due to incorrect identities or algebraic errors being made.

Most students who reached the stage of achieving a value for a suitable trigonometric function successfully applied the inverse, e.g. arccos(1/4) (or equivalent), to get the primary solution of 75.5° and many proceeded to identify the second solution of 284.5°, with only a few adding to 180° instead of subtracting from 360°. The mark scheme condoned the inclusion of the solution 180°, which was fortunate as this was extremely common and only a minority of students observed that this solution was not in the domain. The inclusion of invalid extra solutions was not common, but was penalised.

Question 9

This question was generally answered very well, providing good access late in the paper and marks concentrated towards the higher end of the spectrum overall, with part (c) being the most challenging and discriminatory - this part did earn the full range of marks, but the final mark relatively rare to be scored (about 1 in ten students) as the constraint found in part (a) was usually overlooked.

In part (a) most students were able to identify the asymptote and therefore correctly stated the value of k. There were a few students who gave the answer as an equation of the line $x = -1$, which was accepted. However, 2, −2 or 1 were seen as common wrong answers, while some made no attempt at this part, not appreciating where the ln function is not defined.

Part (b) was well attempted by the majority, better so than part (a). Most students were able to identify the coordinate of A for part (i) by setting $x = 0$ and proceeding to get a correct y coordinate. Anything that indicated the correct coordinate was accepted, for instance $A = 2$ was seen, or $y = 2$, (0,2) or just 2 were all acceptable. Part (b)(ii) was less secure but still well attempted. Most were able to use a valid method for the x intercept and achieved $2-4\ln(x+1)=0$, with the problems that arose coming from how to solve this equation.

Many successfully reached a correct answer of $e^{\frac{1}{2}}-1$, though some wrote $x = 0.65$ rather than an exact answer, losing the A mark. An inability to manipulate logs led to mistakes on this part, and indeed the rest of the question. Some students attempted to expand the bracket when dealing with the log e.g. $\ln(x+1) = \ln x + \ln 1$. This repeated mistake was costly in this question overall. Other common errors seen were $x=\frac{1}{2}e-1$ and $\ln\frac{1}{2}-1$.

Removing the modulus sign when solving part (c) proved more challenging and some solutions only attempted one inequality. The majority, however, did successfully identify the two correct equations to solve and generally went on to successfully find the critical values. Fewer errors were made when solving the equations $\pm\left(2-4\ln(x+1)\right)=3$ than $2-4\ln(x+1)=\pm 3$. Typical errors included reaching $\ln(x+1)=\frac{1}{4}$ (rather than $-\frac{1}{4}$) and $-4\ln(x+1)>1 \Rightarrow \ln(x+1)>-\frac{1}{4}$, though the latter was recoverable in the final answer. A few solutions rejected $\ln(x + 1) = -\frac{1}{4}$. Once again decimal values were seen despite the instruction for exact answers.

Drawing the line $y = 3$ on the diagram to identify the outside region was rarely considered and two marks were often lost at the end as incorrect regions were deduced. Those who identified that the outside region was to be found usually failed to pick up full marks here due to failing to identify the $x > -1$ constraint and incorporate this into their solution, despite the focus of part (a) to draw attention to it.

Typical incorrect answers were $x < e^{-\frac{1}{4}} - 1$ or $x > e^{\frac{5}{4}} - 1$, $x > e^{-\frac{1}{4}} - 1$ or $x > e^{\frac{5}{4}} - 1$ and $e^{-\frac{1}{4}} - 1 < x < e^{\frac{5}{4}} - 1$. Also common as to include all three intervals in the answer, and sign slips in one of the powers. However, it is encouraging to note that the vast majority of students are correctly identifying regions using either set notation or use of inequalities.

Question 10

The majority of students attempted this question, which given that it was the last question of the paper indicated that timing was not an issue on this paper overall. Only about a fifth scored no marks, but these were usually due to incorrect attempts rather than no attempt. A typical total for this question was the first 5 or 6 marks out of a possible 9.

The majority of students were able to access part (a) of the question and successfully identified the *y* coordinate having substituted $x = \frac{1}{4}$ into the expression. However, attempts to solve $\sin^2 4y = \frac{1}{4}$ were often disappointing with a failure to carry out the square root and then apply the arcsin function, with expressions like $\frac{1/4}{\sin 4}$ seen, while some thought the square root of $\frac{1}{4}$ was $\frac{1}{16}$. Some students worked in degrees so 7.5° appeared a number of times rather than $\frac{\pi}{24}$, losing the A mark. Others didn't give an exact answer, giving for example, $y = 0.131$, and these were commonly the ones who went on to score no marks for the question. Extra solutions were often given but were not penalised provided they were outside the domain.

Part (b) was also generally well executed with students differentiating the given expression to obtain an expression for $\frac{dx}{dy}$. Though most were able to achieve a correct form, differentiation of $\sin^2 4y$ to achieve the form $A \sin 4y \cos 4y$ or equivalent did prove challenging for some and expressions such as $\sin^2 4y$, $4\cos y$, $\cos^2 4y$, $8\cos^2 4y$ and $8\cos 4y$ were seen. Most students remembered the "$\frac{dx}{dy} =$" in their answer, but a few did omit this.

Students who achieved the correct form $\frac{dx}{dy} = 4\sin 8y$ often ran into problems in the next part when trying to get the derivative in terms of *x*. There were a very small number of attempts to make *y* the subject and differentiate, as in the alternative method on the mark scheme. These were often incorrect, and generally failed to state $\frac{dx}{dy}$ losing the A mark in any case.

Nearly all students who attempted part (c) achieved the first method mark as they recognised they needed to find the reciprocal of the expression in part (b). The exception tended to be those who made y the subject first to obtain a derivative in terms of *x*, finding $\frac{dy}{dx}$ directly.

Replacing sin 4*y* and cos 4*y* to achieve a value for $\frac{dy}{dx}$ in terms of *x* was not so well done, with either no attempt, or incorrect identities used. An extra square on the *x* from an incorrect $\sqrt{1-\sin^2 4y} = \sqrt{1-x^2}$ was common. Only a small number of students who achieved a correct form were able to go on to write $x - x^2$ in completed square form correctly, again many simply making no attempt. A small number of students achieved an answer in the form with $(2x - 1)$ in the bracket, so not quite the form specified, but these were still able to access marks in (c).

Part (d) was not accessed by many students; indeed the final three marks on this paper were very discriminating. The issues seem to be less with part (d) itself, though, and more to do with the inability to find a suitable answer to part (c). Those had achieved an answer to part (c) of the required form they were actually quite successful in finding the minimum point and the *x* value for where this occurred. Most students had failed to achieve the correct form though and therefore could make no further progress in this question, even though errors in completed square notation were followed through. There were a few cases of suspected use of graphical calculator to obtain the correct answers from incorrect work in part (c), but these were not allowed credit as the question required the use of part (c) to answer the question.